Skirting Hidden-Variable No-Go Theorems

A. F. Kracklauer
Belvederer Allee 23c
99425 Weimar, Germany
kracklau@fossi.uni-weimar.de
(February 1, 2008)

It is observed that the proofs of hidden-variable no-go theorems depend on the 'projection postulate,' which is seen to be contradictory with respect to spin operators in directions orthogonal to the magnetic field direction. In this light it is argued that it is less costly to abandon the projection postulate than to abandon locality and this in turn renders Hidden-Variable No-Go Theorems evadible. To buttress this point, a local realist model of the EPR experiment which ignores the constraints of the projection postulate is presented.

3.65.Bz, 01.70.+w

I. INTRODUCTION

In the Theory of Quantum Mechanics (QM), as is well known, complementary variables can not be determined to arbitrary precision. This fact in combination with certain symmetries, however, leads directly to a contradiction. Consider, for example, the disintegration of a stationary particle into twin daughters moving off in opposite directions. For each daughter separately, only the position or momentum can be measured with arbitrary precision. Now however, if at a given instant the position of one daughter and the momentum of the other are determined precisely, then by symmetry, the complementary position and momentum for both twins can be deduced with arbitrary precision, seemingly in conflict with QM. These considerations clearly imply that in spite of a QM dictum, complementary variables must have a certain 'reality' independent of that which should be created by a measurement.

Indeed, in 1935 Einstein, Podolsky and Rosen (EPR) used exactly this argument to challenge the assumed completeness of QM. EPR anticipated the possibility that the insinuation of additional, heretofore 'hidden,' variables could render QM complete; that is, with the aid of these extra variables the calculation of even complementary quantities could be done, at least in principle, with arbitrary precision. In fact, inspired by the above arguments, EPR used the characteristic of being predictable with certainty as the very definition of 'reality.' [1] Accordingly, attempts to discover these extra, still hypothetical, hidden variables have been identified with a so called 'realist' school.

Since that time Von Neumann [2], Bohm [3], Bell [4] and many many other researchers have with various tacks studied the issue. [5] In spite of, perhaps because of, the essential ambiguity of the interpretation of QM, some sought refuge in the rigor of mathematics. But, within the discipline of Physics it is impossible to prove a theorem. The proof of a theorem requires a set of established theorems and definitions built up from a base of axioms. This is possible in Mathematics and Logic but the Natural Sciences are different. They are programs to find the 'axioms of Nature,' as it were. These axioms certainly are largely unknown (else basic research could be abolished in favor of engineering, etc.), and candidate axioms, in the form of fundamental theories, currently under discussion are not mutually fully consistent. Lacking axioms, there can be no web of syllogisms, hence no proven theorems, just theories with their intrinsically tentative nature.

In this light one may ask just how has Bell's 'Theorem,'—to the effect that efforts by 'realists' to identify 'hidden' variable are doomed to finding only such which are nonlocal; i.e., those entailing dependency on events outside past light cones—been proven? Rummaging in the literature eventually discloses that for this purpose an implicit axiom set has been mandated. As this set comprises the formal structure of QM and not just empirical data or the phenomena to be described, the physical justification for these axioms is open to suspicion. In particular, and this will turn out to be the nub of the argument made herein, it includes the 'projection postulate' or the assertion that an individual measurement on a single system in effect, always and exclusively, 'projects' out the eigenvalue of an operator on an appropriate Hilbert space as a result. Without this postulate, or at least parts of its operational equivalent, proofs of Bell's Theorems do not go through. It will be argued below that imposing this postulate on hidden variable formulations of QM restricts such theories in a way in which the core of QM itself in fact is not. The cost of abandoning this postulate, however, is trivial compared with that of abandoning locality.

As regards a 'hidden variable' or 'realist' program, such an adaptation is in harmony with the motivation of its proponents. Their goal was and is to explain 'Nature' not QM. If this can be done successfully without the full generality of the projection postulate, but otherwise in accord with the empirically verified results of QM, there can be no objection.

That the possibility of evading Bell's results exists was

telegraphed already in 1980 by the late Pierre Claverie who found a mathematical gremlin in the proof. [6] His point is that dichotomic functions representing spin cannot be correlated to yield a harmonic function, which is the so called 'QM-result' (Eq. (14) below). The question of just how general this objection is, and whether it extends to all variations of Bell's Theorem remains open. It is the point of this note to address this issue.

II. SOME FUNDAMENTALS

We recall here some elementary facts concerning spin. Although well known, they are essential below and for convenience are repeated. [7]

Given a uniform static magnetic field $\mathbf{B}$ in the z-direction, the Hamiltonian is:

$$\mathbf{H} = +\frac{e}{mc}\mathbf{B}\sigma_z. \tag{1}$$

For this Hamiltonian, the time-dependent solution for the Schrödinger equation is:

$$\psi(t) = \frac{1}{\sqrt{2}}\begin{bmatrix} e^{\frac{-i\omega t}{2}} \\ e^{\frac{+i\omega t}{2}} \end{bmatrix}, \tag{2}$$

and it gives time-dependent expectation values for spin values in the x, y directions respectively:

$$\langle\sigma_x\rangle = \frac{\hbar}{2}\cos(\omega t),\ \langle\sigma_y\rangle = \frac{\hbar}{2}\sin(\omega t), \tag{3}$$

where $\omega = e\mathbf{B}/mc$.

What is to be seen here is that in the x-y plane, in contrast to the z, magnetic field, direction, expectation values can not be made up out of averaged eigenvalues. In this plane a direct application of the projection postulate seems absurd. In fact, what is usually meant by measuring spin in these directions requires reorienting the magnetic field so that in effect one again is measuring σ_z in the new B-direction.

III. HIDDEN-VARIABLE NO-GO THEOREMS

There are two basic formulations of Bell's hidden-variable no-go theorems: a version found first by Bell [8] but shortly thereafter refined and generalized by Kochen and Specker [9] and a second version found by Bell [10] and subsequently modified by Clauser to accommodate experimental exigencies. [11]

The latter version is the one most often discussed in the literature.

A. Bell-Kochen-Specker Theorems

For the purposes of this note, I call on a version of the Bell-Kochen-Specker Theorem expounded by Mermin. [5] At the onset, the system of interest is presumed prepared in a 'state' $|\psi\rangle$ and described by observables $A, B, C \ldots$. A hidden variable theory is then deemed to be a mapping v of the observables to numerical values: $v(A), v(B), v(C)\ldots$ so that if any observable or mutually commuting subset of observables is measured on that system, the results of the measurements on it will be the appropriate values. Just how the values are fixed is the substance of the particular theory.

Now if this theory is to be compatible with QM, the observables are operators on a Hilbert space (An erudite way to say that the solutions of the Schrördinger Equation reflect its hyperbolic nature.) and that the measured values $v(A)$, etc. are the eigenvalues of these operators. It is simply a fact that if a set of operators all commute, then any function of these operators $f(A, B, C\ldots) = 0$ will also be satisfied by their eigenvalues: $f(v(A), v(B), v(C)\ldots) = 0$.

From this point, the proof of a Kochen-Specker Theorem proceeds by displaying a contradiction. Surely the least complicated rendition of the proof considers two 'spin-1/2' particles. For these two particles, nine separate mutually commuting operators are formed into the following 3 by 3 matrix:

$$\begin{array}{ccc} \sigma_x^1 & \sigma_x^2 & \sigma_x^1\sigma_x^2 \\ \sigma_y^2 & \sigma_y^1 & \sigma_y^1\sigma_y^2 \\ \sigma_x^1\sigma_y^2 & \sigma_x^2\sigma_y^1 & \sigma_z^1\sigma_z^2. \end{array} \tag{4}$$

It is a little exercise in bookkeeping to verify that any assignment of plus and minus ones for each of the factors in each element of this matrix results in a contradiction, namely, the product of all these operators formed row-wise is plus one and the same product formed column-wise is minus one.

Here we see immediately that the proof depends on simultaneously assigning the [eigne]values ± 1 to σ_x, σ_y and σ_z as measurables for each particle. (With little effort, for all other proofs of this theorem one can find the same assumption.) However, in Section II above, we saw that if the eigenvalues ± 1 are believable measurable results in one direction, then in the other two directions the expectation values oscillate. Thus, in the oscillating directions, the projection postulate makes no sense. (It appears as if it should at least be limited to operators for which the matrix of eigenvectors is diagonalized. Moreover, it has been suggested that the projection postulate is generally invalid. [12] This author has great sympathy for this assertion on the grounds that ontologically realizable states should yield positive definite Wigner densities. [13] Such considerations are beyond the limited scope of this note, however.) If the projection postulate is abandoned for these directions, then the proof of a Bell- Kochen-Specker

theorem does not go through. Sacrificing it for x- y spin operators has no practical cost for QM—in fact, it seems necessary for internal consistency. Moreover, I note that this proof of this theorem makes no specific use of the properties of hidden-variables, so that it in principle pertains to QM as currently formulated; it speaks more to the viability of the projection postulate than of hidden variables.

B. Bell's Famous Theorem

The formulation of Bell's more widely known version of a no-go theorem provides constrains that are billed as unavoidable by all realist-local theories and which are violated by QM. Once again, however, the derivation depends on implicit hypothesis including the projection postulate for more than one direction at a time. To show this explicitly, consider Bell's seminal paper [10] where the argument proceeds as follows.

One considers two functions $A(\mathbf{a}, \lambda)$ and $B(\mathbf{b}, \lambda)$ which are to represent measurements, in the case of an EPR experiment, the measurement of the polarization of 'photons' emerging from a process wherein the emitted radiation can carry off no angular momentum; i.e., the photons must be oppositely polarized. As is well known the singlet wave function for this situation is ambiguous with respect to the polarization. In an effort to resolve this ambiguity and invest in the wave function complete predictability (reality), a set of heretofore 'hidden' variables, λ is insinuated. This is the 'reality' stipulation. Further, as these measurements are to be independent of influences outside past light- cones, $A(\mathbf{a}, \lambda)$ is to be independent of $\mathbf{b}$; i.e., variables pertaining to the measuring apparatus at the location of $B(\mathbf{b}, \lambda)$, which likewise is independent of $\mathbf{a}$. This is the 'locality' stipulation.

Finally, each of the functions is allowed to take the values ± 1 to correspond to the supposition that exactly two states of polarization are to be found. This is the way in which the projection postulate is imposed on a hidden variable theory even if this theory in its final form should not make use of operators on a Hilbert space. While this final assumption seems quite innocent, it entails many concealed assumptions as in fact all that can be observed is a photoelectric generated electron, not a distinct 'photon,' for example. As these functions take on the values ± 1 for all directions, e.g., $\mathbf{a}$ and $\mathbf{b}$, our point is actually complete here. But to exhibit precisely how this assumption enters into the derivation of Bell inequalities, I continue.

The correlation of these function then is:

$$\mathbf{P}(\mathbf{a}, \mathbf{b}) = \int d\lambda \rho(\lambda) A(\mathbf{a}, \lambda) B(\mathbf{b}, \lambda), \tag{5}$$

which is asked to duplicate the QM result:

$$\langle \sigma^{\mathbf{1}} \cdot \mathbf{a}\, \sigma^{\mathbf{2}} \cdot \mathbf{b} \rangle = -\mathbf{a} \cdot \mathbf{b}. \tag{6}$$

To derive the famed inequalities, one then uses $A(\mathbf{a}, \lambda) = -B(\mathbf{a}, \lambda)$, in the expression

$$\begin{aligned} &P(\mathbf{a}, \mathbf{b}) - P(\mathbf{a}, \mathbf{c}) = \\ &-\int d\lambda \rho(\lambda) \left[A(\mathbf{a}, \lambda) A(\mathbf{b}, \lambda) - A(\mathbf{a}, \lambda) A(\mathbf{c}, \lambda) \right], \end{aligned} \tag{7}$$

which is then factored

$$\begin{aligned} &P(\mathbf{a}, \mathbf{b}) - P(\mathbf{a}, \mathbf{c}) = \\ &\int d\lambda \rho(\lambda) A(\mathbf{a}, \lambda) A(\mathbf{b}, \lambda) \left[A(\mathbf{b}, \lambda) A(\mathbf{c}, \lambda) - 1 \right], \end{aligned} \tag{8}$$

where explicit use is made of $A(\mathbf{b}, \lambda) A(\mathbf{b}, \lambda) = 1, \forall\, \mathbf{b}$, (i.e, the projection postulate) to get

$$|P(\mathbf{a}, \mathbf{b}) - P(\mathbf{a}, c)| \leq \int d\lambda \rho(\lambda) \left[1 - A(\mathbf{b}, \lambda) A(\mathbf{c}, \lambda) \right], \tag{9}$$

which gives

$$1 + P(\mathbf{b}, \mathbf{c}) \geq |P(\mathbf{a}, \mathbf{b}) - P(\mathbf{a}, \mathbf{c})|, \tag{10}$$

which is one variation of a Bell inequality. Alternate derivations of similar inequalities employ the projection postulate in a similar way.

IV. CORRELATIONS, QUANTUM VS. LOCAL-REALIST

If in fact it is problematic to ascribe dichotomic values to spin in more than one direction simultaneously, the question naturally arises regarding just what the oft performed and empirically verified correlation calculation in QM textbooks accomplishes and how does it do it. The answer to the above question then is that underneath the abstraction and notation of QM, the calculation is actually only a rendition of Malus' Law from classical optics applied to an ensemble of systems. This is best seen by examining the derivation of the formulas used to compare calculations with observations.

To begin, one considers the singlet state for a pair of 'photons' as is usual for analysis of the Einstein-Podolsky-Rosen-Bohm experiment:

$$\psi = \frac{1}{\sqrt{2}} \left[|x_1\rangle |y_2\rangle - |x_2\rangle |y_1\rangle \right], \tag{11}$$

where here the indices x_1, y_1, etc. indicate photons polarized in the x, y direction respectively and propagating away from each other along the z direction. The photons are then analyzed (measured) by passage through, say, Wollaston prisms which separate the polarization states geographically. Of this it can be said: 'They perform dichotomic measurements, i.e., a photon can be found in one of the two exit channels, labeled $+1$ or -1. This is similar to a Stern- Gerlach filter acting on spin 1/2 particles.'

For QM calculations now one transforms this state, Eq. (11), to a rotated frame using the standard rotation matrix:

$$\begin{bmatrix} |x_1'\rangle \\ |y_1'\rangle \end{bmatrix} = \begin{bmatrix} \cos(\phi) & -\sin(\phi) \\ \sin(\phi) & \cos(\phi) \end{bmatrix} \begin{bmatrix} |x_1\rangle \\ |y_1\rangle \end{bmatrix}, \tag{12}$$

so that for ψ in the rotated frame, one gets:

$$\psi' = \frac{1}{\sqrt{2}} [\, \cos(\phi)|x_1'\rangle|y_2'\rangle - \cos(\phi)|y_1'\rangle|x_2'\rangle \\ - \sin(\phi)|x_1'\rangle|x_2'\rangle - \sin(\phi)|y_1'\rangle|y_2'\rangle \,]\,. \tag{13}$$

From this expression, the essentials for QM calculations follow directly. The crosscorrelation is computed as follows:

$$\psi'^* M_1 M_2 \psi' = -\cos(2\phi), \tag{14}$$

where M_1 is a 'measurement' made on photon 1; i.e., $M_1|x_1\rangle = +1|x_1\rangle$, $M_1|y_1\rangle = -1|y_1\rangle$. The probability that both photons are polarized in the x'-direction, for example, is the square of the third term:

$$P(x_1', x_2') = \frac{1}{2}\sin^2(\phi). \tag{15}$$

This is Malus' Law with an extra factor of 1/2. Both the factor of 1/2 and the apparent failure here to conform to the rationale of Malus' Law; i.e., that it is to be the comparison of intensity measurements made in series, distinguish this result from the usual of this law.

A semiclassical, sometimes denoted 'naive,' model of the EPR emissions has been suggested. It takes the detection probability to be proportional to the received field intensity: $(\mathcal{E}\cos(\theta))^2$ where θ is the angle between the polarization direction of the signal and the axis of the polarizer used in the detector. A coincidence detection is proportional to the product of a detection probabilities in each channel. The resultant probability is proportional to: $\cos^2(\theta)\cos^2(\theta-\phi)$ where ϕ is the angle between the axes of the measurement polarizers if the coordinate system is aligned with one of them. (This is just Malus' Law generalized to a frame rotated by θ.) Finally, the total probability is obtained by averaging over many pairs of signals, each with its own randomly given polarization angle θ, that is

$$\frac{1}{\pi}\int_0^{\pi} [\cos(\theta)\cos(\theta-\phi)]^2 d\theta = 1/4 + 1/8cos(2\phi). \tag{16}$$

To convert this intensity to a probability it must be converted to the ratio of coincidence rate divided by the total *pair* production rate. For ideal detectors, the number of detections is linearly proportional to the field intensity so the number of pairs of detection events will be 1/2 the value of the field intensity integrated over the detector apertures—in the above equations, this factor in the local units has been taken to be 1, so that finally Eq. (16) must be divided by 1/2.

This expression does not violate a Bell Inequality. It would be an ideal rebuttal to the conundrums evoked by Bell's [theory] were it to agree with experiment. However, this result has a nonzero minimum whereas Eq. (15) does go to zero and this difference is observable; Eq. (16) does not conform to Nature. [14]

This simple observation would settle any dispute regarding the existence of a local realist alternative to QM were the above semiclassical model exhaustive. In fact it is not—at least not if the projection postulate is abandoned. A different result is obtained if to the above semiclasscial model the following two modifications are made:

a. the source is assumed to be comprised of a cloud of independent point sources (e.g., atoms) which emit spherical radiation into 4π steradians (in their own rest frames) with random phase offsets, γ determined by; *inter alia*, random locations in the cloud, random motion and so on; and,

b. it is assumed that although a two stage cascade emission is facilitated by an intermediate step, that the radiation from each stage alone is unpolarized even if each stage seems to have a distinct frequency. This means that the radiation from each source (atom) received in each detector will comprise two independent random contributions, one each from each polarization state.

The phase difference between each polarization state is a random function of time. The signal received at the detector aligned with the coordinate system is proportional to two terms, one for each polarization state with its own phase offset, $\gamma_{x,y}$:

$$A(\theta, \gamma_x, \gamma_y) = \cos(\theta)\cos(\gamma_x) + \sin(\theta)\cos(\gamma_y), \tag{17}$$

and likewise, the signal at the detector oriented at angle ϕ to the other detector is proportional to:

$$B(\theta, \phi, \gamma_x, \gamma_y) = \\ \cos(\theta+\phi)\cos(\gamma_x) + \sin(\theta+\phi)\cos(\gamma_y). \tag{18}$$

The coincidence count registered by ideal detectors will be then proportional to the average over all incidence angles, θ, of the squared product of these two factors averaged over the phase offsets, $\gamma_{x,y}$:

$$\frac{1}{\pi}\int_0^{\pi} [\frac{1}{\pi^2}\int_0^{\pi}\int_0^{\pi} A(\theta,\gamma_x,\gamma_y)B(\theta,\phi,\gamma_x,\gamma_y)d\gamma_x d\gamma_y]^2 d\theta. \tag{19}$$

This integral, divided again by the ideal *pair* production rate, evaluates to:

$$\frac{1}{2}\sin^2(\phi). \tag{20}$$

Thus, in conclusion, this purely realist-local formulation with erstwhile hidden variables θ, γ_x and γ_y yields

a result which is identical to that from QM. What allows this calculation to go through is that it violates the projection hypothesis by admitting not only more than two values of the field intensity at the detectors, but also values greater than 1, namely the maximum value of $sin(\theta) + cos(\theta)$, i.e., $\sqrt{2}$ as the sum of two orthogonally polarized signals. The factor $\sqrt{2}$ is recognized as that by which QM violates Bell inequalities.

Abandoning a strict application of the projection hypothesis is not new. Bell considered new definitions of the functions $A(\mathbf{a}, \lambda)$ and $B(\mathbf{b}, \lambda)$ which could take on the value zero in addition to ± 1 to take into account failed detections of generated 'particles.' [15] (Zero is not an eigenvalue of any Pauli spin operator.) Then for these new functions a new inequality restricting their averages, $\overline{A}(\mathbf{a}, \lambda)$ and $\overline{B}(\mathbf{b}, \lambda)$, was derived and used to show that a certain class of stochastic local hidden variables are incompatible with QM—given the hypothesis, the essential element of which here is being less than the norm of the eigenvalues of a spin operator: 1.

Likewise, Clauser and Horne reformulated the derivation of Bell inequalities with the goal of obtaining versions that can be compared with experiments. The basic problem they faced was that in order to obtain observable probabilities using the original Bell formulation, it is necessary to determine accurately the total production rate of pairs. However, if it is taken that some 'particles' go undetected, then in principle measuring this rate accurately is impossible. Clauser and Horne evaded this problem by using only ratios of detection rates which are equal to ratios of detection probabilities as the normalizing factor, the total pair generation rate, fortuitously divides out.

In the derivation of these altered inequalities, however, the projection postulate once again tacitly serves as motivation. It enters in the assumption that a count-event is caused by an entity for which a dichotomic function specifies its state. That is, it is assumed that, for example, a count is triggered by a photon with either pure x or y polarization, and not by some combination for which the norm could be greater than 1. The new local realist model described above violates exactly this assumption by assuming that the signal triggering a detection event (ejection of a photoelectric electron) has a contribution from each polarization state.

Of course, the above new local realist model need not be evaluated exclusively under the umbrella of conceptual categories set by the projection postulate. It stands alone as a counterexample to the claim that no local realist model can duplicate QM.

V. LOCALITY AND PROJECTION POSTULATE

There are seemingly deep philosophical considerations regarding locality that, almost uniquely for such topics, are largely instinctively understood by any thoughtful person. However interesting such matters may be, they are not within the purview of Physics which has is own requirements for locality. The latter are expressed as the demand that equations of motion be mathematically well posed.

From this requirement two degrees of 'locality' can be discerned: what can be called 'light induced' locality that requires all causes to be within the past light cone of their effects, and 'general locality' which requires that such causes be within the past light-like cone where the speed of 'interaction' is any thing below the limit $c \to \infty$. Violating general locality, this author maintains [16], results in poorly posed, unintegrable equations of motion and for this reason alone, strictly from within the discipline of physics, must be rejected. In the zone between light and general locality, interactions faster than the speed of light can not be ruled out *a priori.* Of course, until a new force is found for which the interaction speed exceeds that of light, or perhaps the discovery that the speed of gravity exceeds the speed of light, all such considerations are exploratory.

Thus, indications that QM reveals some kind of nonlocal interaction, need not be taken by itself as fundamentally unacceptable if what is meant does not violate general locality. But, "extraordinary claims require extraordinary evidence." Here, the 'extraordinary' cost of admitting the so far otherwise totally unsupported hypothetical existence of a new and superluminal force is much higher than abandoning the at best formalistic projection postulate.

One might ask, exactly what purpose the projection postulate serves. For us today it is difficult to follow in detail the thought sequences of the originators, but it seems possible that they were focused on fitting distinct spectral lines into operators on Hilbert Spaces. [17] The effective assumption appears to have been that multiple lines seen when analyzing the emissions from a particular source arise separately from distinct atoms. This imagery has become enshrined in the notion that each atomic transition results in a distinct photon.

However, the appearance of distinct spectral lines results whenever an emission results from a constrained system for which the relevant equations have compact support. A guitar string, for example, when struck will generally not vibrate at a pure tone, be it the fundamental or any harmonic, but rather at some combination. Spectral analysis, nevertheless, will reveal distinct lines—but one is not entitled to conclude that the signal is emitted by an ensemble of guitars, each sending a pure tone. For all the same reasons, atomic spectra can not be attributed to the sum of pure signals from individual transitions in an atom so that an unrestricted application of the projection postulate is unwarranted, rendering the postulate itself questionable if not downright inconsistent. Of course, reconciling new, alternate imagery with 70 years of precedence will take thought and time.

VI. CONCLUSIONS

When fully understood, what Bell's Theorems do do is prove just what their hypothesis allows, namely that a reformulation of QM with hidden variables will be unable to parameterize particle spin with dichotomic functions in directions different from the magnetic field. At the heart of this issue is the tacit assumption that particles have dichotomic spin in arbitrary directions intrinsically, regardless of external magnetic fields, which, when introduced by observers, just 'measure' the preexisting spin in the direction of the field. If, on the other hand, spin is seen as gyration actually engendered by the magnetic field, this whole issue and its attendant confusion do not arise.

What these theorems do not do, however, is generally preclude hidden variable formulations. Likewise, if the projection postulate is sacrificed in whole or part, they do not invest any part of Physics with nonlocal relationships of any kind, including for correlations.

[1] A. Einstein, B. Podolsky and N. Rosen; Phys. Rev., **47**, 777 (1935).

[2] J. von Neumann, *Mathematische Grundlagen der Quantenmechanik*, (Springer Verlag, Berlin 1932). By a querk of history, the no-go proof found in this book can be seen as a 'nonlocal' anticipation of the problem faced in [1]. It was dispatched in [8].

[3] D. Bohm, Phys. Rev., **85**, 180 (1952).

[4] J. S. Bell, *Speakable and Unspeakable in Quantum Mechanics*, Cambridge University Press, Cambridge (1987).

[5] For a current review and bibliography see: N. D. Mermin, Rev. Mod. Phys. **65**(3), 803 (1993); and D. Home and F. Selleri, Rivista del Nuovo Cimento, **14**(9) (1991).

[6] P. Claverie and S. Diner, Israeli J. Chem. **19**, 54 (1980). See: A. F. Kracklauer in *New Developments on Fundamental Problems in Quantum Physics*, M. Ferrero and A. van der Merwe (eds.), (Kluwer Academic Publishers, 1997), p.185 for brief analysis of the same point.

[7] R. H. Dicke and J. P. Wittke, *Introduction to Quantum Mechanics*, (Addison-Wesley, Reading 1960) p. 195.

[8] J. S. Bell, Rev. Mod. Phys. **38**, 447. Reprinted in [4].

[9] S. Kochen and E. P. Specker, J. Math. Mech. **17**, 59 (1967).

[10] J. S. Bell, Physics **1**, 195 (1964). Reprinted in [4].

[11] J. F. Clauser and M. A. Horne, Phys. Rev. D, **10**(2), 526 (1974).

[12] M. Ferrero and E. Santos, Found. Phys. **27**(6) 765 (1997).

[13] A. F. Kracklauer, Phys. Essays, **5**(2) 226 (1992).

[14] In: J. F. Clauser, Phys. Rev. A **6**(1), 49 (1972), this model in its full version to accomodate nonideal detectors under the rubric of 'Semiclassical Theory' was analyzed and compared with experiment. I thank C. H. Thompson, author of: "The Chaotic Ball," Found. Phys. Lett. **9**, 357 (1996); for setting me straight on many aspects of semiclassical models.

[15] J. S. Bell, *Proc. of the Int. School of Physics 'Enrico Fermi', course IL*, (Academic Press, New York, 1971), p.171. Reprinted in [4].

[16] A. F. Kracklauer, J. Math. Phys. **19**(4), 808 (1978).

[17] R. I. G. Hughes, *The Structure and Interpretation of Quantum Mechanics*, (Harvard University Press, Cambridge, 1989) p. 217.